Exploring Mental Computation in the Middle Years

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Following earlier interest in mental computation and its relationship to number sense in the early and primary years, this exploratory study investigated the mental computation competence of students in the middle school. The study involved clustering the responses of four students in grades 6 and 8 based on individual interviews. In each grade a student was identified as more or less competent in mental computation performance on a pencil-and-paper class test. Several contrasting content and performance features were identified and examined with respect to implications for teaching mental computation in the classroom.

In an educational climate that endorses the development of a flexible and integrated approach to numeracy, mental computation is recognised for its educational and its utilitarian value. Interest in the "ability to calculate exact numerical answers without the aid of calculating or recording devices" (Reys, Reys, & Hope, 1993, p. 306) is by no means new. However, redirecting the practice of mental computation in the classroom away from the instant recall of rehearsed facts to incorporate computational strategies and calculations larger than those covered by basic number facts is new. Mental computation research has mainly focused on the early and primary years of schooling yet in light of the current attention to numeracy in the middle years (Siemon, 2001), there seems reason to examine what mental computation involves at this level.

The middle years of school represent a period where the scope of the curriculum expands and children begin to reason abstractly (Watson & Callingham, 2001). In mathematics, students incorporate their understanding of number and place value to embrace relational concepts, measurement and space (Siemon, 2001). Similarly, as the curriculum demands increase during this period, the span of student achievement also becomes increasingly differentiated (Hill, Rowe, Holmes-Smith, & Russell, 1996). In terms of its educational value, mental computation skill is linked with number sense and is posited to encourage a creative approach to solving problems and facilitate students' understanding of number properties and operation (McIntosh, De Nardi, & Swan, 1994). Yet Weber (1996) contends that middle grade students do not appear to regard numbers as quantities with multiple internal relationships, but rather as symbols to manipulate. What then happens to the mental computation abilities of students as they make the transition into middle school?

Mental computation research has rarely extended beyond year 6 (or year 7 depending on the primary school classification system) to incorporate the early years of secondary school. Large-scale studies that do include these middle grades tend to do so within a range of grades, providing only snapshots of how middle grade students perform (McIntosh, Nohda, Reys, & Reys, 1995; Reys, et al., 1993). One such Australian study explored performance across grades 3, 5, 7, and 9. The authors, McIntosh, Bana, and Farrell (1995), considered three perspectives of mental computation including preference, attitude, and performance assessment. This research however provided an overview of quantitative data that did not extend to cover strategy choice or to detail related cognitive components of students' thinking.

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In terms of qualitative data, individual interview-based research has been invaluable in advocating mental computation as a valid computational method and one which contributes to mathematical thinking as a whole (Sowder, 1990). In-depth interviews have led to descriptions of strategies that students utilise to solve mental computation problems, particularly addition and subtraction (McIntosh, et al. 1994; Heirdsfield, 2000). Interestingly strategy use can be described as idiosyncratic and self-taught (Bana & Korbosky, 1995), with younger children particularly employing spontaneous strategies before direct teaching (Heirdsfield, 2000).

Mental computation interviews have also been a rich source of information in terms of looking at the relationships among factors that mediate mental computation acquisition and use. A complex set of connections involving number sense, cognitive factors (specifically memory), affective factors, and metacognitive processes have been identified in the literature. Predominantly these factors have been combined to look at accuracy in mental computation (Heirdsfield, 2001). Perhaps the most frequently linked factor with mental computation is number sense (McIntosh & Dole, 2000). Two findings in particular with primary students raise some interesting questions for the middle school level. First, good mental computation ability may not necessarily be supported by well developed number sense (McIntosh & Dole, 2000). Second, strategies start to reflect pen-and-paper algorithm procedures as influenced by instruction towards the end of primary school (Heirdsfield, Cooper, & Irons, 1999). These findings, observed in interviews with upper primary students, seem relevant in examining mental computation in the middle school sector and suggest many research questions. The one under consideration in this paper is: What features distinguish middle school students at different competence levels?

The research reported in this paper constitutes an aspect of an on-going mental computation research project currently being conducted in Tasmania and the ACT. The mental computation interviews from which data were extracted for the study form the basis for documenting the variety and type of strategy use to be expected at successive developmental levels.

Method

Subjects

Students in grades 3-10 at a primary and a secondary school in the Tasmanian Catholic sector completed a paper-and-pencil mental computation class test (Callingham & McIntosh, 2002). Three students in each grade (3-10) were selected for interview as representatives of their grades based on three ranges of raw data scores: 50%-60%, 70%-75%, and >85%. From the 12 middle school students (grades 5-8) interviewed, four students were chosen for case study analysis, two from each of grades 6 and 8. These students were two low scoring males (raw score 50%-60%, one in each grade), and a high scoring male and a female (raw score >85%, one in each grade). They were also assigned a Rasch mental computation competence level within the range of 1 to 8 (Callingham & McIntosh, 2002). In grade 6 the low and high scoring students were level 5 and 7/8 respectively and in grade 8 the low and high scoring students were level 6 and 8.

Protocol

Initially a set of base interview questions was selected from the paper-and-pencil mental computation tests (Callingham & McIntosh, 2002). These questions were selected to incorporate all whole number operations with some decimal, percent, and fraction questions mainly for the secondary students. Each base question was then extended into a set of related questions (for example see Table 1). It was this set of questions that was presented to students during the interviews.

Table 1
Example of an Interview Question Set

24 x 3	Base question from test	Double-digit multiplication question
24 x 6	Related question	Related double-digit multiplication problem in the same form
24 x 30	Related question	Related double-digit multiplication problem with a multiple of ten

Interview Procedure

The students were interviewed individually in a separate room in their respective schools. The interview sessions were approximately 30 minutes for both the primary and the secondary students and were videotaped with parental permission. The interview protocol was semi-structured in that the questions were specifically constructed and a number of potential directions were anticipated depending on how the students responded. Nevertheless, the interview was still guided by individual characteristics, responses, and comments from the students. Participation in these interviews was voluntary and the students were told they could conclude the session any time they desired. None stopped early.

Data Analysis

A cyclic clustering procedure (Miles & Huberman, 1994) was employed using the responses and discussion recorded for the four case study students. A case-by-attribute matrix was the result, with each row representing a question asked in the interview and the responses being distributed over the attribute columns. Initial clustering of columns was informed through observation and with reference to a framework for examining number sense (McIntosh, Reys & Reys, 1992) and a framework for proficient mental computation (Heirdsfield, 2001). A second part to this process involved discussion among three members of the research team to identify common characteristics among the rows and columns of the matrix. Overall six features were identified as distinguishing the more competent students from the less competent students in their mental computation interview performance.

For the following discussion the four students will be identified based on their raw scores and Rasch levels as: L6 (less competent grade 6), M6 (more competent grade 6), L8 (less competent grade 8), and M8 (more competent grade 8).

Results

Each of the six features identified as distinguishing students of different mental computation competence will be presented with examples from the interviews where appropriate. These examples will be presented in the following format: the identified student and the question (both in italics), and then the student response. Additional questions by the interviewer will also be in italics. The observations are considered in three content features, two performance features, and a feature concerning making connections.

Use of Elementary Number Work

Strategies that the students used to solve mental problems showed that in some cases the less competent students relied greatly on aspects of their elementary number work. In solving the multiplication questions, for example, the less competent students used repeated addition strategies to solve mental problems.

Student L8 (Question 24 x 3). 72. I went up 24 + 24 + 24.

The more competent students however, used strategies that reflected an understanding of place value.

Student M8 (Question 24 x 3). 3 x one 4 is 12, 3 x one 20 is 60, plus them together is 72.

This facility with place value also enabled these students to make more calculated decisions for whether to use either the units first or the tens first, depending on the properties of the operation. The use of repeated addition is related to the use of doubling in solving multiplication problems. Instances of doubling were noted among the less competent students. Again, it could be argued that this strategy reflects the influence of elementary number work from the early primary years.

Extended Number Facts

The questions based on extended number facts provided another feature where the more competent and less competent students displayed some differences. The term "add a zero" was employed by both categories of students when multiplying by ten (or twenty in one case) and when dividing a number by a multiple of ten. However the more competent students seemed to utilise this to a greater extent and were able to adapt their 'zero' rules to solve the division question. The facility with this rule for these students, in that zeros were purposely added and removed, was reflected by the wide range of expressions used to describe this strategy, for example, 'dissect the ten into zeros', 'put the zero on', and 'cancel down the zeros'.

Student M8 (Question 7 x 30 (presented after 7 x 3)). 210. Just add a zero *(because?)* because it's 10s. *(Question 70 x 30)* 2100. Just add two zeros. *(Question 420 / 70 (presented after 42 / 7)).* 6. Because they're both 10s so you cancel down the zeros.

In contrast the less competent students did not rely on the 'zero' rule. Student L8 identified two ways in which to solve the problem 12 x 10. The first method involved 'adding the zero to the 12 from the 10', whereas the second method involved the 12 being split into 10 x 2 and 10 x 10. On the subsequent problem, 22 x 10, student L8 acknowledged that he would 'add a zero' in this case as the other way was 'too hard'. Student L6 only 'added a zero' once and in this instance he answered incorrectly.

Student L6 (Question 70 x 30 (presented after 7 x 3)). 210. 3 x 7 is 21, add the zero.

Fractions and Decimals

When dealing with fraction and decimal questions there were a number of observable differences between the more competent and less competent students. Primarily the less competent students tended to translate fraction questions into decimals. As illustrated by the examples below, $\frac{3}{4}$ became .75 and $\frac{1}{2}$ became .50. One student in particular indicated a preference for thinking in decimals. With the two less competent students, one answered in decimals, the other in fractions, but both explanations were translated into language associated with decimals.

Student L6 (Question $\frac{3}{4}-\frac{1}{2}$). 25. You just take away 50 which equals 0.25.

Student L8 (Question $\frac{3}{4}-\frac{1}{2}$). $\frac{3}{4}$ is like 75, 50 is a $\frac{1}{2}$, take away the 50 is 25 so $\frac{1}{4}$.

In contrast the more competent students demonstrated facility in moving easily between fraction and decimal representations. There seemed to be a clear boundary separating the two domains, with answers and explanations given in the appropriate terminology, respectively.

Student M6 (Question $\frac{3}{4}-\frac{1}{2}$). $\frac{1}{4}$ 2/4 is a $\frac{1}{2}$ so take that away.

The tendency for the less competent students to apply their decimal knowledge to fraction questions is interesting and it raises several points for consideration. First, the translation from fractions to decimals in one sense makes it a complicated way to solve what is not a particularly difficult question for students at the middle school level. Being able to recognise that $\frac{1}{2}$ is the same as $\frac{2}{4}$ appears a much easier route to solving $\frac{3}{4}-\frac{1}{2}$. Second, the translation is useful only in solving a handful of fraction questions. When given the question $\frac{1}{2}-\frac{1}{3}$ one of the less competent students demonstrated that he had no available means by which to proceed with this question.

Student L8 (Question $\frac{1}{2}-\frac{1}{3}$). 2.5...1/3 is 75. I don't know. *(Do you know what $\frac{1}{3}$ of 100 is?)* 35?...40?

Errors

A feature of these individual interviews in comparison with the timed tests completed earlier is that students were given the opportunity to discover and rectify their own errors during discussion. Overall, the students appeared to be differentiated by the way they managed errors. Less competent students seemed less inclined to check their answers, at times seeking assurance from the interviewer. There were more instances when the less competent students gave initial incorrect answers but rectified their responses during the explanation as to how they worked out the answer.

Student L8 (Question 7 x 6). 51? 41. 7 + 7 = 14, + 14 is 28. Add another 14 to get...42.

In addition the less competent students had to be prompted through their own explanations to assist them in identifying an answer. When given the opportunity to attempt a related question with an incorrect strategy, one of the less competent students completed the question but did not acknowledge the error. The confidence displayed by these students seemed to be at a much lower than for the more competent students.

Student L6 (Question 125–99). 24. Take away 100 first, 25 and then take the 1. *(Question 135–99).* 34 *(Did you use the same strategy?)* Yes.

Overall, the more competent students made very few errors. If in doubt they seemed to recheck again before confirming they were satisfied with their answers. It appeared from the demeanour of these students that those errors that did occur could be classed as a type of 'slip up' error due to speed rather than a conceptual error. The more competent grade 6 student when given two harder questions that she could not complete (1.2 x 10 and $\frac{1}{2}-\frac{1}{3}$), acknowledged the difficulty she was having yet persisted in working through the question with the interviewer.

Timing

During the interviews and when watching the video tapes the speed at which the more competent students processed the questions and produced an answer was striking compared to the less competent students. Whereas the more competent students appeared to process several components of the information concurrently, the less competent students appeared more laboured in decomposing and recomposing the information to arrive at an answer. Subsequently these students answered fewer questions overall and with less coherent explanations in the interview session as to how they arrived at their answers.

Making Connections

The sets of related questions were constructed specifically to explore the information students recognised and exploited from related questions. Providing the first question in the set was answered correctly, the students had the opportunity to either: recognise and use the previous question to assist them in solving a new problem without having to start the computation process from the beginning or, recognise that a previous strategy was successful and apply it again (although it could be argued that a combination of both was also possible). Whereas all students recognised the similarities in subtracting 7 (or 70) from numbers that ended with 7 (or 70), the less competent students could not respond similarly to harder questions. Both less competent students had successful strategies to solve 100–55 but struggled to adapt their strategies to answer a related question, 100–34.

> *Student L8 (Question 100–55).* 45. Go from 55, add 40 which adds up to 95, then you add the 5 which equals 100. *(Question 100–34).* 76 (student asks the interviewer if this is right and decides that it's wrong and starts again). 66. Worked it out using my hands. Have your 60, add the 30 so 90, 4 on your 34, add 6 to that and get 100.

The more competent students, on the other hand appeared to move without difficulty from 100–55 to 100–34.

> *Student M8 (Question 100–55).* 45. Took away 50 from 100 and took away another 5. *(Question 100–34).* 66. 30 (take) 100 is 70, take another 4.

Examples from the more competent students emphasise that these students brought to their mental computation performance an appreciation of number that facilitated making connections. It is possible to see the students utilise both a successful previous strategy and the number information from a related question.

> *Student M6 (Question 125–99).* I took the 25 from the 99 which is 74. Then I took the 74 from the 100 which is 26. *(Why did you take 25 from 99?).* Because it's easier to work in 100s so we have to take the 25 off the 125 so we can work in the 100s. OK, so we're left with the 74 so we take the 74 out of the 100 and that leaves us with 26. *(Question 135–99)* 36. *(Did you do that from the last answer).* Yes. I remembered that the last answer was 26 so then I was working with a bigger number so I just add the 10. *(Question 125–89).* 36. Sort of the same thing but it's a different number. I knew that 99 out of 125 was 26 and we were working with a different number so the total is going to be bigger because the number was smaller and I knew it was bigger by 10.

> *Student M8 (Question 125–99).* 26. 99 to 100 is 1 so I just plussed the 25. *(Question 125–89).* 36. Because there's 11 up to 100 and plus the 25. *(Question 125–79).* 46. It's 21 up to 100 and plus the 25.

Discussion and Implications

This was an exploratory investigation impelled by the question, what features distinguish students of different mental computation competence at the middle school level? Although it is not possible to generalise from the results of four individual students, there were some features that differentiated the more competent and less competent students, and some interesting points for discussion about teaching mental computation at the middle school level.

In terms of elementary number work the less competent students used repeated addition and some related doubling strategies, whereas the more competent students preferred place value strategies. The latter may reflect a familiarity with place value as influenced by pen-and-paper procedures during the upper primary years (Heirdsfield, et al. 1999). Although a repeated addition strategy reflects that students implicitly understand the properties of the operation they are using, continuing to rely on these strategies may prevent students from being able to broaden the scope of the problems they can manage, particularly for multiplication and division problems. Similarly, for the less competent students the use of 'adding zeros' with extended number facts appeared to be utilized but not necessarily understood well. Zeros appeared to be regarded as independent entities from the rest of the number by all students but the more competent students were still better equipped to manipulate with understanding, the role zeros play in performing division operations. The role of teaching practices for these two features involves setting up environments where students will be exposed to the range of strategies for a particular sort of problem, and learn to value the process of explaining answers and sharing strategies. Heirdsfield, et al. (1999) showed in a longitudinal case study that 'adding zeros' was used early in year 4 so it is likely this is a strategy that has been explicitly taught in the classroom.

The fact that the less competent students translated fraction questions into language more closely associated with decimals was an interesting finding of this study. When investigating the effects of instructional materials on mental computation procedures, Weber (1996) described 8th grade students' understanding of fractions, decimals, and percents as limited, in that conceptual knowledge bases were inadequate. It would appear in the present study that while all the students had some strategies to solve basic decimal problems, the less competent students did not respond similarly when faced with fractions. Conversely the more competent students were able to access a much greater range of fraction problems. Part-whole number relationships constitute a greater part of the curriculum at the middle school level than in the earlier years. Since the inter-relatedness of part-whole numbers (fractions, decimals, or percents) is essential in establishing conceptual understanding, and in being able to solve these problems mentally, teachers have a role to build clear representations of both using the appropriate terminology.

The cautious way that the less competent students dealt with errors, along with their lack of inclination to check answers in the interviews, seems to be an issue related to confidence. Connections between mental computation and perceptions of ability have been identified in proficient mental computers (Heirdsfield, 2001). The observation in the present study that the more competent students appeared to process several components of information concurrently verifying Heirdsfield's findings regarding the function of memory in supporting and rehearsing interim calculations.

The sets of related questions provided an encouraging avenue in exploring the types of information students recognise and subsequently use to formulate a computational plan. Although it was often difficult to obtain an 'uninterrupted' set of questions, the potential of this type of questioning will be further examined in research particularly with regard to how it might be employed in the classroom. Using related questions may assist in making students more aware of alternatives when choosing a strategy. It might be the case that a well-rehearsed strategy may not necessarily be appropriate or the easiest to use; helping students to learn to discriminate between related questions may prove a useful exercise.

This study identified some features that appear to distinguish the competence of the mental computation performances for four middle school students. The transition into middle school will be the subject of ongoing research particularly in establishing the role of mental computation practices within the middle school curriculum.

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